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Numerical modelling of three-dimensional fluid flow in a spiral compensator

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Abstract

The problem of the numerical modelling of the damping effect of the spiral compensators of percussion–rotary drilling devices is considered. The Roe first-order difference method has been adapted for the computation of a three-dimensional flow of a barotropic compressible fluid on a spatial curvilinear grid. As a result, the distributions of the solution components have been obtained both inside the compensator channel and at its upper outlet. A comparison of the damping effect of the compensator with the results of the one-dimensional computation by a TVD second-order scheme has shown that the three-dimensional computations produce a slightly more pronounced damping effect than the one-dimensional computations.

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1. Introduction

The percussion–rotary technique is currently the main technique for the drilling of wells in the sedimentary rocks for the exploratory purposes and oil and natural gas production. This technique is inefficient

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Fig. 1. The basic elements of the percussion–rotary drilling device: (a) the hydraulic drilling hammer; (b) spiral compensator; (c) the curvilinear grid on the internal walls of a spiral channel for $N_1 = 161$, $N_2 = 12$, $N_3 = 12$. The dimensions are given in meters.

at the drilling in hard crystalline rocks (Pixton, 1990; Zhao, 1998). The percussion–rotary drilling is a combination of the rotary and percussion methods and has the following advantages over the pure rotary drilling:

- higher drilling velocity in moderately hard and hard stone;
- better stability of well direction and
- insignificant wear because of lesser rotation frequency and lesser drilling pressure.

The percussion–rotary drilling technique was implemented in the hydraulic drilling hammers of several types (Pixton, 1990; Zhao, 1998). Following Schacht et al. (2002), we now describe in detail the design and the functioning of a typical hydraulic drilling hammer and the spiral compensator. The hydraulic drilling hammer (Fig. 1(a)) consists of the shock piston (1), anvil (2) and drill bit (3) as well as a valve (4) with a spring (5).

The working fluid (the distilled water) enters from above via the valve (4), which moves depending on the volume rate against the spring strength downwards, and then the fluid flows through the shock piston (1), which has a larger frontal surface at the lower fluid outlet than at the inlet, into the space I, via the channel in the anvil. Since the lower frontal surface of the shock piston (1) is larger than its upper frontal surface, a larger pressure develops on the lower frontal surface than on the upper frontal surface of the piston. Because of this difference in pressures the shock piston (1) moves upwards and finally leads to a sudden interruption of flow when its upper frontal surface encounters the valve (4). This results in a pressure shock (also called shock wave), which accelerates the valve (4) motion as well as the downward motion of shock piston (1). Then it again frees, however, the flow at the lower dead point of the valve (4). The piston (1) hits due to its inertia the anvil (2), and the above process repeats again.

When the valve (4) is closed the shock wave forms above the valve, which then propagates in the spiral compensator (shown in Fig. 1(b)) upwards. After the shock wave already damped in the spiral compensator has left the spiral compensator it propagates further upwards in a pipeline and enters a reservoir which is used as a pressure accumulator of the pump. Thus, the shock wave propagates in the spiral compensator in the direction opposite to the fluid flow. The purpose of the spiral compensator is to damp the periodical shock waves, which could destruct both the drilling device and the pipeline supplying the fluid to the percussion–rotary drill.

Fig. 1(b) shows a typical actual spiral compensator of the drilling unit; the compensator diameter is $4\frac{3}{4}$ in, and the drilling unit produces the borehole 6 in in diameter.

We now explain the way in which the fluid flows through the drilling device. Inside the drilling hammer the fluid flows downwards. The drill bit (3) is supplied with inclined nozzles through which the fluid leaves the drilling hammer and then flows, together with stone crumb produced as a result of drilling, upwards. The inclined positioning of the nozzles in the drill bit ensures that the drill bit (3) turns by a certain angle (percussion–rotary drilling) after each impact of the piston (1) against the anvil (2). Due to the high frequency of the impacts, the mixture of fluid and stone crumb flows rather continuously upwards (6). The path of the fluid (which subsequently carries the crumb) is shown in Fig. 1(a) by the dashed lines supplied with arrows.

Thus, the formation of shock waves is a feature of the fluid flow inside the percussion–rotary drilling device. The pressure behind the fronts of these shocks may take very large values. The spiral compensators are designed in such a way that they must reduce maximally the strength of shock waves and to maximally use the shocks propagating in the direction of anvil and drill bit for the drilling processes.

The mathematical modelling of time-dependent hydrodynamic processes in spiral compensators is of great practical importance for an optimal design of such pressure compensators. The hydrodynamic processes in a spiral compensator were modelled by Schacht et al. (2002) on the basis of the equations of the model of an inviscid compressible barotropic fluid in the one-dimensional approximation. These equations were approximated by two different finite difference schemes: the scheme of splitting in terms of physical processes and a TVD scheme. The numerous numerical computations by these schemes have enabled us to find the optimal geometric configurations of the spiral compensators, which ensure the best efficiency of the damping of shocks.

The one-dimensional formulation, however, does not enable the consideration of the curvature effects of the spiral compensator walls as well as the centrifugal forces. In this connection we propose below an extension of the mathematical model of fluid flow applied by Schacht et al. (2002) for the case of a three-dimensional time-dependent compressible fluid flow in a spiral compensator.

We now present a review of a number of works in which the finite difference methods were applied for the numerical modelling of three-dimensional compressible flows of liquids or gases in the channels with curved walls and with a rectangular cross section. Ferziger and Perič (2002) have presented the results of the numerical solution of the problem of a three-dimensional viscous laminar flow around a cylinder mounted in a channel with a rectangular cross section and with straight walls.

Aubert et al. (1995) have applied the first- and second-order schemes for the numerical modelling of a three-dimensional transonic flow in a nozzle whose geometry included a plane channel with 10% sinusoid bumps on the upper and lower walls. Zha and Bilgen (1996) have proposed an implicit difference scheme

for the computation of a three-dimensional flow in an experimental ONERA channel, which had three planar walls and a bump on the lower wall. The three-dimensional flow in this channel was characterized by the presence of a system of oblique shocks reflecting from the channel walls.

There are also experimental studies of the shock reflections from the curved channel walls; a review of the relevant works was made by Bazhenova et al. (1986), Aki (1989), and Fursenko et al. (1992).

It is to be noted that there are at least two significant differences between the problems considered by Bazhenova et al. (1986), Aki (1989), Fursenko et al. (1992), and the problem of the shock propagation in a spiral compensator:

- the shock Mach number in a spiral compensator does not exceed 0.03, so that the three-dimensional flow which is considered in the following belongs to a class of low Mach number flows;
- the curvature of the spiral compensator walls does not undergo any jump in contrast with the case considered by Bazhenova et al. (1986), Aki (1989), Fursenko et al. (1992), Aubert et al. (1995), Zha and Bilgen (1996), and Ferziger and Perič (2002).

As was pointed out by Bazhenova et al. (1986), a specific shock wave configuration arising at the reflection of shocks from the walls is determined completely by the incident shock strength. If the angle between the shock front and the interaction surface exceeds 90°, then a shock diffraction process occurs, which is characterized by the formation of rarefaction waves in the gas flow behind the shock wave (Bazhenova et al., 1986). Such a situation arises in the problems considered by Bazhenova et al. (1986), Aki (1989), Fursenko et al. (1992), Aubert et al. (1995), Zha and Bilgen (1996), and Ferziger and Perič (2002), when an initially planar shock wave front reaches the point of matching the rectilinear and curved parts of the channel walls.

In our case the vertical channel walls have everywhere the same constant curvature, and the angle between the initially planar shock wave front and the channel walls equals 90°. Thus, the conditions for the formation of the shock diffraction phenomenon are not satisfied in our case. This results in a much simpler flow pattern than in cases of shock diffraction problems considered by Bazhenova et al. (1986), Aki (1989), Fursenko et al. (1992), Aubert et al. (1995), Zha and Bilgen (1996), and Ferziger and Perič (2002). The fluid dynamics problem considered in the following is nevertheless computationally very intensive. This is conditioned by the following factors: (i) a complex geometry of the spatial computational region; (ii) the necessity of the execution of several thousands of time steps to ensure a reliable prediction of the maximum values of the fluid pressure at the upper outlet of the spiral compensator.

A shock wave flow regime is realized in the existing designs of spiral compensators. Schacht et al. (2002) have proposed an idea of replacing this regime by a smooth flow regime by means of installing an extra valve supported by a spring at the lower inlet of the compensator. The spring strength must be adjusted in such way that a sufficient smoothing of pressure profiles in time be ensured. It was shown by Schacht et al. (2002) that in the case of a smooth pressure function at the lower inlet of the compensator. In addition, the damping effect of the compensator has been shown to be more considerable for the same pressure amplitude at the lower inlet of the compensator than in the shock wave regime.

The conservative difference schemes are applicable for the computation of both smooth and discontinuous solutions of the fluid dynamics problems. Volpe (1993) has considered several popular computer codes (implementing the methods of the Runge–Kutta type) as applied to the computation of two-dimensional flows with the free stream Mach number $M_{\infty} = 0.003$. A conclusion was drawn on the applicability of the

computer codes also for the computations at such low Mach numbers. This conclusion was stimulating for us at the choice of one of currently popular schemes, the Roe scheme (Roe, 1981; Toro, 1999), for the computation of a three-dimensional nonstationary flow of a compressible inviscid barotropic fluid in a spiral compensator.

The paper is organized as follows. In Section 2, we present the governing equations for three-dimensional barotropic compressible fluid flow in curvilinear coordinates. For a better understanding of the obtained numerical solutions, and for a validation of this solution, it is very important to know at least some features of the exact solution. In this connection, we study in Section 3 some properties of the solution of the three-dimensional problem under consideration. In Section 4, we briefly describe the Roe scheme as applied to three-dimensional barotropic fluid flows.

Since we have at present no results of the experimental measurements in actual spiral compensators at our disposal, we have used for the verification of the developed numerical method the test problem of the stationary shock wave propagation in a channel with flat walls. For this purpose, the original spiral channel was rectified to a one-dimensional channel in Section 5 by means of simple formulas for the corresponding mapping of the region in the physical spatial coordinates onto a similar region with rectangular boundaries in the space of curvilinear coordinates. We present further in Section 5 the results of the numerical modelling of three-dimensional hydrodynamic processes in actual spiral compensators. And, finally, in Section 6 we formulate the conclusions.

2. Governing equations

As the flow model we will use the Euler equations governing a three-dimensional nonstationary flow of an inviscid compressible nonheat-conducting barotropic fluid. Denote by Ω the spatial region inside a spiral compensator. Since the spiral channel walls are curved it is more convenient to use the equations of fluid flow in curvilinear coordinates ξ , η , ζ such that the original region Ω in the space of the Cartesian spatial coordinates x, y, and z is mapped onto a parallelogram Π in the space of curvilinear coordinates. The ξ coordinate is measured along the spiral, whereas the η and ζ coordinates in the cross section. Assume that there is the corresponding one-to-one mapping

$$x = x(\xi, \eta, \zeta), \quad y = y(\xi, \eta, \zeta), \quad z = z(\xi, \eta, \zeta), \quad (\xi, \eta, \zeta) \in \Pi.$$

$$\tag{1}$$

The flow equations in curvilinear coordinates ξ , η , and ζ take the following form (Pulliam and Steger, 1980):

$$\frac{\partial \mathbf{U}J}{\partial t} + \frac{\partial \hat{\mathbf{F}}}{\partial \xi} + \frac{\partial \hat{\mathbf{G}}}{\partial \eta} + \frac{\partial \hat{\mathbf{H}}}{\partial \zeta} = J\mathbf{R}(\mathbf{U}).$$
(2)

Here, *J* is the Jacobian of transformation (1), $J = x_{\xi}(y_{\eta}z_{\zeta} - y_{\zeta}z_{\eta}) - y_{\xi}(x_{\eta}z_{\zeta} - x_{\zeta}z_{\eta}) + z_{\xi}(x_{\eta}y_{\zeta} - y_{\eta}x_{\zeta});$ $x_{\xi}, y_{\xi}, z_{\xi}, x_{\eta}, y_{\eta}, z_{\eta}, x_{\zeta}, y_{\zeta}, z_{\zeta}$ are the partial derivatives, for example, $z_{\eta} = \partial z(\xi, \eta, \zeta)/\partial \eta$, etc.;

$$\hat{\mathbf{F}} = J\xi_x\mathbf{F} + J\xi_y\mathbf{G} + J\xi_z\mathbf{H}, \quad \hat{\mathbf{G}} = J\eta_x\mathbf{F} + J\eta_y\mathbf{G} + J\eta_z\mathbf{H}, \quad \hat{\mathbf{H}} = J\zeta_x\mathbf{F} + J\zeta_y\mathbf{G} + J\zeta_z\mathbf{H}, \quad (3)$$



Fig. 2. The rectangular cross section of a spiral channel.

where the column vectors U, F, G, and H are defined by formulas

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \end{pmatrix}, \quad \mathbf{F}(\mathbf{U}) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \end{pmatrix}, \quad \mathbf{G}(\mathbf{U}) = \begin{pmatrix} \rho v \\ \rho u v \\ \rho v v \\ \rho v^2 + p \end{pmatrix}, \quad \mathbf{H}(\mathbf{U}) = \begin{pmatrix} \rho w \\ \rho u w \\ \rho v w \\ \rho v w \\ \rho w^2 + p \end{pmatrix},$$
$$\mathbf{R}(\mathbf{U}) = \begin{pmatrix} 0 \\ -\lambda \rho |\vec{v}| u/(2D) \\ -\lambda \rho |\vec{v}| v/(2D) \\ -\lambda \rho |\vec{v}| w/(2D) - \rho g \end{pmatrix}. \tag{4}$$

In Eqs. (4) ρ is the fluid density; $\vec{v} = (u, v, w)$ is the vector of fluid velocity, where u, v, w are the components of the velocity vector along the *x*-, *y*-, and *z*-axis, respectively; *p* is the pressure, and \vec{g} is the vector of acceleration due to gravity (9.81 m/s²) and $|\vec{v}|$ is the modulus of the velocity vector, that is, $|\vec{v}| = \sqrt{u^2 + v^2 + w^2}$.

To close Eq. (2) the equation of state for fluids

$$p = K_{\rm eff} \, \ln(\rho/\rho_0) \tag{5}$$

is needed. The quantity ρ_0 in (5) is the initial fluid density at t = 0. The constant K_{eff} , which is also known in literature as bulk modulus, may be computed for a rectangular channel as follows (Jenkner, 1971; Schacht et al., 2002, 2005):

$$K_{\rm eff} = [E_{\rm F}^{-1} + 0.1(B/H)^4 R(\beta)(d/H)^{-3} E_{\rm M}^{-1}]^{-1},$$
(6)

where E_F and E_M are the Young's modules of elasticity for fluid and wall material, respectively, *d* is the wall thickness, and *B* or *H* are the external dimensions of the channel cross section (see Fig. 2). The quantity $R(\beta)$ is the so-called rectangular factor:

$$R(\beta) = (6 - 5\beta)/2 + (1/2)(H/B)^5(6 - 5\beta(B/H)^2), \quad \beta = 1 - H/B + (H/B)^2.$$
(7)

The dimensional quantities p, ρ , u, v, and w are related to the nondimensional quantities \bar{p} , $\bar{\rho}$, \bar{u} , \bar{v} , \bar{w} by formulas

$$\bar{p} = \frac{p}{K_{\rm eff}}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{u} = \frac{u}{c_0}, \quad \bar{v} = \frac{v}{c_0}, \quad \bar{w} = \frac{w}{c_0}, \quad \bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{L}, \quad \bar{z} = \frac{z}{L},$$

$$\bar{t} = \frac{t}{t_{\rm ref}}, \quad c_0 = \sqrt{\frac{K_{\rm eff}}{\rho_0}},$$
(8)



Fig. 3. The friction factor λ in the interval $10^5 \le Re \le 10^6$ computed by formula (9) (dashed line) and by the Nikuradse formula (10) (solid line).

where *L* is the length of the spiral channel and $t_{ref} = L/c_0$. We have omitted the bars over the nondimensional quantities in (4) and in the following for the purpose of brevity of notation. In the nondimensional form, the equation of state (5) takes the form: $\bar{p} = \ln \bar{\rho}$.

The term $-\rho\lambda |\vec{v}|\vec{v}/(2D)$ in (4) takes into account the losses due to wall friction in channel. A number of different computational formulas for the wall friction number λ were proposed by Abramovich (1991) for the case of a stationary flow as functions of the Reynolds number and the relative wall roughness. Because of the periodic shock waves the flow in a spiral rectangular channel is strongly nonstationary. Therefore, we have used in the present work the following relations basing on the experimental data:

$$D = B_1 H_1 / (B_1 + H_1), \quad \lambda = 0.021 D^{-0.3}$$
⁽⁹⁾

for the computation of the effective diameter D and the wall friction factor λ (Merenkov et al., 1992). Here, B_1 and H_1 are the internal dimensions of flow channel (see Fig. 2) in a cross section, which is perpendicular to the channel walls.

Basing on the input data presented in Section 6.2.1 it is easy to find that for typical conditions of water flow in spiral compensator the Reynolds number varies from 10^5 to 10^6 . One can, therefore, use also the formula of Nikuradse (Warsi, 1999) for the friction factor λ :

$$\lambda = 0.0032 + 0.221 \, Re^{-0.237},\tag{10}$$

where $Re = \bar{q}D/v$, \bar{q} is the mean flow velocity in the duct cross section, and v is the fluid kinematic viscosity. If, for example, $B_1 = 0.06 \text{ m}$, $H_1 = 0.0266 \text{ m}$, the volume flow rate $Q_0 = 600 \text{ l/min}$, $v = 10^{-6} \text{ m}^2/\text{s}$, then we obtain Re = 115, 473. We show in Fig. 3 the graphs of the functions (9) and (10). Formula (9) can be seen to yield the values, which are by a factor of about 3.5 higher than the values obtained by the Nikuradse formula (10). This is not surprising because formula (9) takes into account the experimental data for the ducts, which are affected both by wall roughness and near-wall turbulence. Since in the case of formula (10) the value of λ depends weakly on Re in the Re range under consideration, we have used in our three-dimensional computations the constant value of λ based on the Reynolds number computed from the user-specified volume flow rate Q_0 .

We now introduce at every point $(x, y, z) \in \Omega$ a system of rectangular Cartesian coordinates $\tilde{x}, \tilde{y}, \tilde{z}$ such that the \tilde{x} -axis is directed along a tangent to the central line of the spiral, and the \tilde{y} - and \tilde{z} -axis are oriented in such a way that the coordinate system $\tilde{x}, \tilde{y}, \tilde{z}$ forms a right coordinate system, see Fig. 4.



Fig. 4. The local axis \tilde{x} , \tilde{y} , \tilde{z} in the channel.

Denote by $\vec{l} = (l_1, l_2, l_3)$, the unit vector of a tangent to the \tilde{x} -axis with coordinates l_1, l_2, l_3 in the system of the original Cartesian coordinates x, y, z. The initial conditions may then be written as follows:

$$\bar{p}(\tilde{x}, \tilde{y}, \tilde{z}, 0) = 0; \quad \bar{u} = q^0 l_1, \quad \bar{v} = q^0 l_2, \quad \bar{w} = q^0 l_3; \quad \bar{\rho}(\tilde{x}, \tilde{y}, \tilde{z}, 0) = 1 \quad (\tilde{x}, \tilde{y}, \tilde{z}) \in \Omega,$$
(11)

where q^0 is the dimensionless fluid velocity, which is obtained from the desired volume rate Q_0 and the hydraulic cross-section area $A = B_1 H_1$: $q^0 = Q_0/(Ac_0)$.

As the boundary condition at the inlet of the spiral compensator, that is, at its lower side in the direction of the percussion–rotary drill, two variants for the pressure function $p_0(t)$ are considered:

1. Smooth periodic function

$$p_0(t) = 0.5[p_{\min} + p_{\max} + (p_{\max} - p_{\min})\sin(\bar{\omega}t - 0.5\pi)].$$
(12)

2. Periodic jump function

$$p_0(t) = 0.5[p_{\min} + p_{\max} + (p_{\max} - p_{\min})\operatorname{sign}(\sin(\bar{\omega}t - 0.5\pi))].$$
(13)

The values of p_{\min} and p_{\max} ($0 < p_{\min} < p_{\max}$), that is, the minimum and maximum pressure values at the inlet of spiral channel, depend on the specific type of the percussion–rotary drill (for example, $p_{\min} = 0.2$ MPa and $p_{\max} = 30$ MPa). In Eqs. (12) and (13), $\bar{\omega}$ is the angular velocity, $\bar{\omega} = 2\pi f$, and f is the frequency of pressure oscillations (for example, 30 Hz).

The boundary conditions at the upper outlet of the spiral are determined depending on the flow character at that end, see further details in Schacht et al. (2005).

On the lower and upper walls as well as on the inner and outer walls, it is assumed that the surface is impermeable, i.e., $v_n = 0$, where v_n is the normal velocity component with respect to the wall. Due to the small curvature of the spiral, it is also assumed that $\partial p / \partial n = 0$ on all walls.

Applying the methods of analytic geometry it is not difficult to show that the desired mapping (3) has the following form for the case of a spiral channel:

$$x = (B_1\eta + a - 0.5B_1)\cos\xi, \quad y = -(B_1\eta + a - 0.5B_1)\sin\xi, \quad z = H_1\zeta + c_1\xi - 0.5H_1,$$

$$s_{\min} \leqslant \xi \leqslant s_{\max}, \quad 0 \leqslant \eta \leqslant 1, \quad 0 \leqslant \zeta \leqslant 1.$$
(14)

Here, *a* is the radius of the cylinder on whose surface the spiral channel axis lies; the constant c_1 is determined by the technological constraints for the spiral compensator design (Schacht et al., 2002). The quantity \bar{H}_1 entering (14) is related to the vertical size H_1 of the hydraulic cross section (see Fig. 2) by the formula: $\bar{H}_1 = H_1 / \sin \alpha$.

3. Solution properties

The system of equations (2) governing a three-dimensional nonstationary fluid flow has a complex nonlinear character. It is nevertheless possible to establish some properties of the solution of this system for the case of the specific problem of barotropic fluid flow.

1. *The signs of the eigenvalues of the Jacobi matrices*. In the subregions of smooth flow the system of equations (2) is equivalent to the following nondivergence system:

$$\frac{\partial \mathbf{U}J}{\partial t} + \hat{A}(\mathbf{U},\xi,\eta,\zeta)\frac{\partial \mathbf{U}}{\partial\xi} + \hat{B}(\mathbf{U},\xi,\eta,\zeta)\frac{\partial \mathbf{U}}{\partial\eta} + \hat{C}(\mathbf{U},\xi,\eta,\zeta)\frac{\partial \mathbf{U}}{\partial\zeta} = J\mathbf{R}(\mathbf{U}),$$
(15)

where

$$\hat{\mathbf{A}}(\mathbf{U},\xi,\eta,\zeta) = \frac{\partial \hat{\mathbf{F}}(\mathbf{U})}{\partial \mathbf{U}} = J(\xi_x \mathbf{A} + \xi_y \mathbf{B}), \quad \hat{\mathbf{B}}(\mathbf{U},\xi,\eta,\zeta) = \frac{\partial \hat{\mathbf{G}}(\mathbf{U})}{\partial \mathbf{U}} = J(\eta_x \mathbf{A} + \eta_y \mathbf{B}),$$
$$\hat{\mathbf{C}}(\mathbf{U},\xi,\eta,\zeta) = \frac{\partial \hat{\mathbf{H}}(\mathbf{U})}{\partial \mathbf{U}} = J(\zeta_x \mathbf{A} + \zeta_y \mathbf{B} + \zeta_z \mathbf{C})$$
(16)

and

$$\mathbf{A}(\mathbf{U}) = \frac{\partial \mathbf{F}(\mathbf{U})}{\partial \mathbf{U}}, \quad \mathbf{B}(\mathbf{U}) = \frac{\partial \mathbf{G}(\mathbf{U})}{\partial \mathbf{U}}, \quad \mathbf{C}(\mathbf{U}) = \frac{\partial \mathbf{H}(\mathbf{U})}{\partial \mathbf{U}}.$$
 (17)

It is convenient for the following to introduce the notation:

$$\hat{u} = \xi_x u + \xi_y v, \quad \hat{v} = \eta_x u + \eta_y v, \quad \hat{w} = \zeta_x u + \zeta_y v + \zeta_z w.$$
(18)

The eigenvalues $\lambda_{\mu}^{(\nu)}$, $\nu = 1, 2, 3, \mu = 1, 2, 3, 4$ of the matrices $\hat{\mathbf{A}}$ ($\nu = 1$), $\hat{\mathbf{B}}$ ($\nu = 2$), $\hat{\mathbf{C}}$ ($\nu = 3$) are expressed by formulas presented in Appendix.

We at first analyze the sign of the quantity \hat{u} . Let $U_0 = \sqrt{u^2 + v^2}$, $q = \sqrt{u^2 + v^2 + w^2}$. It is clear that $U_0 \leq q$. We can write the quantities u and v in the form: $u = U_0 \cos \beta$, $v = U_0 \sin \beta$, where $\cos \beta = u/\sqrt{u^2 + v^2}$. Then

$$\lambda_1^{(1)} = \lambda_2^{(1)} = \hat{u} = U_0(\xi_x \cos\beta + \xi_y \sin\beta) = \sqrt{\xi_x^2 + \xi_y^2} U_0 \cos(\beta - \beta_1), \tag{19}$$

where $\cos \beta_1 = \xi_x / |\nabla \xi|$. A typical spiral compensator contains several spiral turns. Since the fluid flow is directed near the vertical walls of the spiral along a tangent to the wall, then (19) implies that the eigenvalues $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ may change their sign within the spatial computational region. One can easily arrive at the same conclusion while considering similarly the eigenvalues $\lambda_1^{(2)}$, $\lambda_2^{(2)}$, and $\lambda_1^{(3)}$, $\lambda_2^{(3)}$.

We now show that always $\lambda_3^{(\nu)} < 0$ and $\lambda_4^{(\nu)} > 0$ for $\nu = 1, 2, 3$. Indeed,

$$\lambda_3^{(1)} = c\sqrt{\xi_x^2 + \xi_y^2} [(U_0/c)\cos(\beta - \beta_1) - 1] \leqslant c\sqrt{\xi_x^2 + \xi_y^2} (M - 1) < 0,$$

where M = q/c is the Mach number, M < 0.1 in the fluid dynamics problem under study. Similarly,

$$\lambda_4^{(1)} = c |\nabla \xi| [(U_0/c) \cos(\beta - \beta_1) + 1] \ge c |\nabla \xi| (1 - (U_0/c)) \ge c |\nabla \xi| (1 - M) > 0.$$

2. The constancy of the density and pressure in the region ahead of the first shock wave front. Denote by $\Lambda_{\xi\eta\zeta}$ a difference approximation of the spatial differentiation operator in (2), that is,

$$\Lambda_{\xi\eta\zeta}\mathbf{U}(\xi,\eta,\zeta,t)\approx-\frac{\partial\hat{\mathbf{F}}}{\partial\xi}-\frac{\partial\hat{\mathbf{G}}}{\partial\eta}-\frac{\partial\hat{\mathbf{H}}}{\partial\zeta}$$

We have used below a finite difference scheme

$$(1/\tau)(\mathbf{U}^{n+1} - \mathbf{U}^n) = \Lambda_{\zeta\eta\zeta}\mathbf{U}^n + J\mathbf{R}(\mathbf{U}^n),$$
(20)

where τ is the time step of the difference scheme, *n* is the number of a time step, $n = 0, 1, 2, \ldots$. Let us find the expressions for \hat{u} , \hat{v} , \hat{w} at t = 0 by using the initial conditions (11) and formulas (18). We also note that the components l_1, l_2, l_3 of the unit tangent vector to the lines of the family $\eta = \text{const}$, $\zeta = \text{const}$ may easily be found with the aid of (14): $l_1 = -(d_1/d_2) \sin \xi$, $l_2 = -(d_1/d_2) \cos \xi$, $l_3 = c_1/d_2$, where $d_1 = a + B_1 (\eta - 0.5)$, $d_2 = (d_1^2 + c_1^2)^{0.5}$. Then we obtain from (18) and (11):

$$\hat{u}(\xi,\eta,\zeta,0) = \xi_x u + \xi_y v = q^0(\xi_x l_1 + \xi_y l_2) = (q^0/d_2)(\sin^2 \xi + \cos^2 \xi) = q^0/d_2.$$

It is then easy to obtain with regard for the formula for the Jacobian *J* that $(\partial/\partial\xi)(J\rho\hat{u}) = 0$. We find in a similar way: $\hat{v}(\xi, \eta, \zeta, 0) = \eta_x u + \eta_y v = q^0(\eta_x l_1 + \eta_y l_2) = [q^0 d_1/(B_1 d_2)](-\cos\xi\sin\xi + \sin\xi\cos\xi) = 0$, therefore, $\partial/\partial\eta[J\rho\hat{v}(\xi, \eta, \zeta, 0)]=0$. And, finally, $\hat{w}(\xi, \eta, \zeta, 0)=\zeta_x u+\zeta_y v+\zeta_z w=[q^0 c_1/(\bar{H}_1 d_2)](-\sin^2\xi - \cos^2\xi + 1) = 0$, so that $\partial/\partial\zeta[J\rho\hat{w}(\xi, \eta, \zeta, 0)] = 0$. Then it follows from (20) that $\rho^1 = \rho^0 = 1$ in the entire spatial region ahead of the first shock wave. By virtue of the fact that the liquid is assumed to be barotropic we obtain that also the pressure *p* remains constant at $t = \tau$ in the above spatial region. Assuming further sequentially n = 1, 2, ... in (20), we arrive at the conclusion that the liquid pressure and density remain constant at the (ξ, η, ζ) points ahead of the first shock wave front in some interval of time until that moment of time when the front of the first shock wave reaches the given point. In order for the first shock wave moving from the bottom to the top of the spiral channel to reach the upper outlet of the channel it is necessary to execute several dozens of thousands of time steps by an explicit difference scheme (cf. also Schacht et al., 2002). In this connection, it is very important for the difference scheme to possess the property: $\Lambda_{\xi\eta\zeta}\mathbf{U}^0 = 0$.

4. Numerical method

Before discretizing the governing equations it is necessary to generate numerically the spatial curvilinear grid inside the spiral channel. This was done by using the analytic formulas (14). Fig. 1(c) gives an idea of the generated orthogonal curvilinear grid. The presented spiral compensator can be seen from this figure to have 12 rolls, so that about 13 grid nodes along the ξ -axis lie within each single roll. The obtained grid on the internal surface may be seen to be insufficiently smooth, so that it is desirable to take a larger number than 161 of nodes along the ξ -axis for the given compensator.

The Roe's scheme (Roe, 1981) has gained a widespread acceptance at the numerical solution of applied fluid mechanics problems; a review of the relevant works may be found in Toro (1999). We will not present here the detailed difference equations of the Roe's scheme for the sake of brevity. We present the Jordan decompositions of the Jacobi matrices needed by the scheme in Appendix.

The expressions for the Roe's averages of the velocity components and density proved to be the same as in the case of the ideal gas equation of state. The Roe's average $\bar{c}_{i+1/2}$ of the sound velocity was found for the case of the equation of state $\bar{p} = \ln \bar{p}$ to have the form (Schacht et al., 2005)

$$\bar{c}_{i+1/2} = ((p_{i+1,j,k} - p_{i,j,k})/\delta\rho_i)^{0.5}, \quad |\delta\rho_i| > \varepsilon; \quad \bar{c}_{i+1/2} = \rho_{i,j,k}^{-0.5}, \quad |\delta\rho_i| < \varepsilon,$$

where $\delta \rho_i = \rho_{i+1,j,k} - \rho_{i,j,k}$, $\varepsilon = 10^{-12}$.

5. Numerical results

5.1. The validation test

In order to check the correctness of the programming of difference equations on curvilinear grid we have used the developed Fortran code in the particular case of a channel with constant cross section, which extends along the *x*-axis. For this particular case, we have used the following transformation from the Cartesian coordinates *x*, *y*, *z* to the curvilinear coordinates ξ , η , ζ : $x = \xi$, $y = B_1\eta$, $z = H_1\zeta$, where B_1 and H_1 are the internal dimensions of the channel cross section x = const, see Fig. 2. Then the metric derivatives obviously simplify to the following expressions: $\xi_x = 1$, $\xi_y = \xi_z = 0$; $\eta_x = \eta_z = 0$, $\eta_y = 1/B_1$; $\zeta_x = \zeta_y = 0$, $\zeta_z = 1/H_1$. To validate the developed computer code we have compared the numerical results with the exact analytic solution for the case of the propagation of a stationary shock wave in a channel. The analytic solution for this test was derived by Schacht et al. (2002).

The numerical results as well as the exact solution (dashed line) are shown in Fig. 5 for the moment of time $t = t_n$, when the shock wave has not yet left the channel. In this run, we have used the spatial uniform grid of $100 \times 10 \times 10$ cells. It is seen from Fig. 5(e), (f) that the velocity of the shock wave propagation is reproduced correctly by the Roe's scheme. The difference solution profiles are monotonous.

As regards the computer code validation via three-dimensional computations. We have found in Section 3 a stationary solution of the Euler equations for the spiral duct, which is characterized by the sine shapes of the velocity components u and v and a constant pressure. This solution should be valid at any (x, y, z) point, which was not reached by the first shock wave. This solution was indeed ensured by the above described Roe's scheme, and the difference solution errors did not exceed the level of the machine roundoff errors at the computations with double precision.

5.2. Flow processes in spiral compensators

5.2.1. Input data for three-dimensional computations

The purpose of studying the three-dimensional fluid flows in actual spiral compensators of the percussion–rotary drilling devices is the determination of such geometric parameters of a spiral channel at which a maximum damping effect is ensured for the periodic shock waves propagating along the spiral channel upwards. With regard for our previous one-dimensional results we have formulated the following

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Fig. 5. Stationary shock wave in channel for the Courant number C = 0.5: the surfaces p = p(x, y, 0, t) (a) and u = u(x, y, 0, t) (b) at the 150th time step; the pressure p(x, 0.02, 0.02, t) (c) and velocity component u(x, 0.02, 0.02, t) (d) at the 150th time step.

goals of three-dimensional numerical computations of fluid flows in spiral compensators:

- the check-up and refinement of the damping effect of a compensator;
- obtaining the information about the flow patterns at different moments of time and in different sections of the spiral channel;
- a comparison of the results of one-dimensional and three-dimensional computations of the pressure at the upper outlet of the compensator channel.

The geometry of a spiral channel with a rectangular cross section is a function of the parameters $a, b, H, d, z_{\text{max}}, \Delta z$. With regard for the conclusions obtained on the basis of one-dimensional computations we have considered in the three-dimensional computations whose results are presented in the following, only the case of a dense packing of spiral rolls to which the value $\Delta z/\bar{H} = 1$ corresponds.

Denote by p_{out} the maximum fluid pressure in the upper outlet section $\xi = s_{max}$ of the spiral compensator. This pressure value was shown by Schacht et al. (2002) to depend strongly on the fluid volume rate Q_0 . In addition, the value p_{out} generally depends on the Young's modules E_F and E_M in accordance with the equation of state. And, finally, the specific damping effect was shown by the one-dimensional computations to be dependent on the pressure values p_{min} and p_{max} in the boundary condition for the pressure (12) or (13) at the lower compensator inlet $\xi = 0$. The outlet pressure value p_{out} is, thus, a function of the following parameters:

$$p_{\text{out}} = F(a, B, H, d, z_{\text{max}}, p_{\text{min}}, p_{\text{max}}, Q_0, E_{\text{F}}, E_{\text{M}}),$$
(21)

Table 1

The values of the pressure p_{out} at the use of the boundary condition (12) for several increasing values of the number N_1 of grid nodes along the ξ -axis

N ₁	161	241	321
p_{out} (bar)	143.60	147.28	148.93
δp_{out}		0.025	0.011

where z_{max} is the spiral compensator height along the *z*-axis. Following Schacht et al. (2002), we have fixed the following parameters in our numerical studies:

$E_{\rm F} = 20.5 \times 10^8 {\rm N/m^2},$	$E_{\rm M} = 200 \times 10^9 {\rm N/m^2},$	$d = 1 \mathrm{cm},$
$z_{\rm max} = 0.6{\rm m},$	$a = 13 \mathrm{cm},$	$\rho_0 = 1100 \text{kg/m}^3$,
$p_{\min} = 1 \text{ MPa},$	$p_{\rm max} = 16 \mathrm{MPa},$	the frequency $f = 25$ Hz.

As a result, function (21) reduces to a function of only three arguments: $p_{out} = F_1(B, H, Q_0)$.

Note that we have taken a shorter compensator than in Schacht et al. (2002) for the three-dimensional computations described below: $z_{max} = 0.6 \text{ m}$. (the value $z_{max} = 0.9 \text{ m}$. was used in one-dimensional computations). This is related to the fact that the number of spiral compensator rolls is proportional to z_{max} according to (14). Therefore, in order to ensure a sufficiently high accuracy of numerical results it is necessary to take a larger and larger number of grid nodes N_1 along the ξ -axis with increasing z_{max} .

5.2.2. The smooth solution case

We at first consider the case when the smooth boundary condition (12) is specified at the lower inlet $\xi = s_{\min}$. As was shown by Schacht et al. (2002) the fluid flow remained smooth in this case during the entire computation and was represented by a sequence of alternating compression and expansion zones. Since the linear size of the spiral channel under consideration along the ξ -axis, that is, its length, is by a factor of about 100 larger than the sizes B_1 and H_1 of its cross section, it is then clear that the choice of the step size $\Delta \xi = (s_{\text{max}} - s_{\text{min}})/(N_1 - 1)$ is the main factor affecting the accuracy of the numerical solution obtained by the Roe's scheme. A grid refinement study was done to verify the result independence. Denote by $(p_{out})_1$ and $(p_{out})_2$ the values of a maximum in the temporal distribution of fluid pressure at the upper outlet of the compensator, which have been obtained for some values $N_1 = N_1^{(1)}$ and $N_1 = N_1^{(2)}$, $N_1^{(2)} > N_1^{(1)}$. We assume the value $N_1^{(2)}$ to be acceptable from the viewpoint of the accuracy of computations by the Roe's scheme if $\delta p_{out} = |(p_{out})_1 - (p_{out})_2|/(p_{out})_2 \cong 0.01$. We present in Table 1 the values of p_{out} obtained for three values of N_1 and for $N_2 = N_3 = 12$ for the moment of time t = 0.03597 s. The value of the volume rate Q_0 equalled 600 l/min. in these computations. The following optimal parameters B and H were found by Schacht et al. (2002) for this value of Q_0 : B = 0.08 m, H = 0.0466 m, which were just used in the above computations. Table 1 implies that the value $N_1 = 321$ is acceptable from the viewpoint of the chosen accuracy criterion. For all values of N_1 from Table 1 30,000 time steps were required to reach the moment of time t = 0.03597 s. The spatial computing grid is shown partially for the case $N_1 = 161$, $N_2 = N_3 = 12$ in Fig. 1(c). In our computations of both smooth and shocked flow regimes we used the grids with $241 \times 12 \times 12$ and $321 \times 12 \times 12$ nodes. For a graphical display of the results along the spiral channel of the compensator this channel was unrolled along a straight line in most figures.

Fig. 6 shows the pressure and the velocity component *u* surfaces in section $\zeta = 0.5$ for two different moments of time $t = 30,000\tau = 0.03597$ s and $t = 37,000\tau = 0.04437$ s. The solution can be seen to



Fig. 6. The pressure surfaces $p = p(\xi, \eta, 0.5, t)$ (a) and the velocity component $u = u(\xi, \eta, 0.5, t)$ (b) at the moments of time t = 0.03597 s (a), (b) and t = 0.04437 s (c), (d). The volume rate $Q_0 = 600 \text{ l/min}$, B = 0.08 m, H = 0.0466 m.



Fig. 7. The profiles of the velocity components u, v, w along the axis $s = \zeta$ in section $\eta = 0.5$, $\zeta = 0.5$ at t = 0.03597 s (a) and at t = 0.04437 s (b): u (solid line), v (dashed line), w (dotted line).

remain smooth also at these times. In Fig. 6 as well as in Figs. 10–13, 17, 20, 21, 22, the horizontal coordinate is the dimensional length of the channel arc measured in meters along the ξ -axis. It can be seen that along the η -axis, the quantity p (and hence ρ) varies weakly. The Cartesian velocity components $u(\xi, \eta, 0.5)$ and $v(\xi, \eta, 0.5)$ can be seen in Figs. 6 and 7 to have the shape of modulated sine curves.

We show in Fig. 8 (the upper row of figures) the streamlines in different sections $\xi = \text{const}$ plotted on the basis of the radial velocity component $u_r = u \cos \xi - v \sin \xi$ and vertical velocity component w. The density contours in different channel sections are presented in the lower row of figures in Fig. 8. Note that a local Cartesian coordinate system $\bar{O}\bar{r}\bar{z}$, whose origin (0, 0) lies in the lower corner of the section $\xi = \text{const}$ nearest to the *Oz*-axis has been used in Fig. 8 as well as in Fig. 18; $\bar{r} = \sqrt{x^2 + y^2} - (a - 0.5B_1)$ is the radial coordinate, $\bar{z} = z - (c_1\xi_i - 0.5\bar{H}_1)$, that is, the coordinates \bar{r} and \bar{z} vary within the following intervals: $0 \le \bar{r} \le B_1$, $0 \le \bar{z} \le \bar{H}_1$.



Fig. 8. Streamlines and density contours in sections $\xi = \text{const}$ at $t = 3 \times 10^4 \tau = 0.03597 \text{ s}$: (a) $\xi = 0.5\Delta\xi$ (the section near the lower inlet); (b) $\xi = 0.5(s_{\min} + s_{\max})$ (the section in the spiral middle); (c) $\xi = s_{\max} - 0.5\Delta\xi$ (the section near the upper outlet).



Fig. 9. The stagnation surfaces q = 0 at different moments of time: (a) $t = 10^4 \tau = 0.01199$ s; (b) $t = 3 \times 10^4 \tau = 0.03597$ s; (c) $t = 33000\tau = 0.03957$ s.

Since the compression wave pushes the fluid particles upwards in the direction opposite to the direction of injecting the fluid via the upper outlet, a stagnation surface must exist at times when the compression wave propagates inside the spiral channel. In order to obtain a more detailed information about this surface we have written a computer code to detect the (x, y, z) points of the surface on which the velocity module $q = (u^2 + v^2 + w^2)^{0.5} = 0$. We show in Fig. 9 the surfaces q = 0 at different moments of time. For convenience of visualization we show in Fig. 9 along with the stagnation surface also the inner vertical wall of the spiral hydraulic channel. The surface q = 0 can be seen to have the shape of a ribbon, which is practically vertical in each section $\xi = \text{const.}$ For small values of time *t*, the extent of the surface q = 0 along the circumferential coordinate ξ is considerable so that it takes more than 2π radians. With increasing *t*, the stagnation surface moves upwards in the direction of the upper outlet of the compensator, and its extent along the ξ coordinate reduces. The latter feature can also be seen in Fig. 10. On different sides of the stagnation surface, the fluid particles not only perform an ascending or descending motion but also have the velocity magnitudes, which depend significantly on the polar radius



Fig. 10. The velocity module surfaces for t = 0.01199 s (a) and t = 0.03597 s (b).



Fig. 11. The tangential velocity component $u_t = u_t(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.03597 s (b).



Fig. 12. The radial velocity component $u_r = u_r(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.03597 s (b).

(see Fig. 9). This results in a considerable extent of the stagnation zone along the spiral axis. All of this evidences a very complex three-dimensional character of fluid motion inside the spiral channel.

In order to gain a further insight into the temporal behavior of the stagnation surface we show in Fig. 10 the velocity module surfaces $q = q(\xi, \eta, 0.5)$ for the same moments of time as in Fig. 9. It can be seen that the points of the surface q = 0, which are located near the outer vertical wall of the spiral channel, for small values of time lag behind the points of this surface, which lie near the inner wall. Therefore, the surface q = 0 wraps around the spiral axis O_z as ξ increases, and simultaneously it approaches the inner channel wall.

We show in Fig. 11 the fluid velocity component $u_t = (ux_{\xi} + vy_{\xi} + wz_{\xi})(x_{\xi}^2 + y_{\xi}^2 + z_{\xi}^2)^{-0.5}$, which is tangent to the grid lines $\eta = \text{const}$, $\zeta = \text{const}$ (these lines run in parallel with vertical channel walls). We also show in Fig. 12 the radial velocity component u_r . Comparing these two figures we can see that the radial velocity component is about two orders of magnitude smaller than the tangential velocity component u_t . This means that the fluid moves in the spiral channel mainly in parallel with the vertical channel walls. It is clearly seen in Fig. 11(a) that the fluid particles move upwards in the spiral channel in the compression



Fig. 13. The surfaces $k_{cf} = k_{cf}(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.03597 s (b).



Fig. 14. The numerical results for the pressure in the case $Q_0 = 600 \, \text{l/min}$ in section $\eta = 0.5$, $\zeta = 0.5$: (a) the temporal pressure distribution in the inlet section $\xi = s_{\text{min}}$ according to (12); (b) the pressure distributions along the spiral channel at different moments of time: $t = 23,000\tau = 0.02758 \,\text{s}$ (solid line); $t = 30,000\tau = 0.03597 \,\text{s}$ (dashed line); $t = 37,000\tau = 0.04437 \,\text{s}$ (dotted line).

wave (the u_t values are positive). Since the compression wave is followed by an expansion wave, the fluid particles behind the compression wave again run in the direction of the lower compensator inlet (Fig. 11(b)). Along with the radial velocity component u_r one can identify in each cross section $\xi = \text{const}$ also the vertical velocity component, which coincides with the Cartesian velocity component w. The component w can be seen in Fig. 7 to be very small.

Each fluid particle rotates around the spiral axis (the *z*-axis) at speed $v_{\xi} = -u \sin \xi - v \cos \xi$. The quantity $\rho v_{\xi}^2/r$, where *r* is the polar radius, $r = (x^2 + y^2)^{0.5}$, is the centrifugal force developed by a unit fluid volume. In order to estimate the relative magnitude of centrifugal force let us introduce the following dimensionless parameter:

$$k_{\rm cf} = \rho v_{\xi}^2 / \left(r \max_{x, y, z} |\nabla p| \right).$$
⁽²²⁾

The surfaces $k_{cf} = k_{cf}(\xi, \eta, 0.5)$ presented in Fig. 13 show that the influence of the centrifugal forces is not large enough to alter the pressure distribution significantly.

Some numerical results obtained for the case of smooth pressure function (12) at the spiral inlet are presented in Fig. 14. The three-dimensional flow in the spiral channel can be seen to remain smooth at different moments of time.

A comparison of the results of a three-dimensional computation of the mean pressure and the mean velocity component u_t in the outlet section of the spiral channel $\xi = s_{max}$ with the results of one-dimensional computation by the second-order TVD scheme from Schacht et al. (2002) is presented in Fig. 15. The mean pressure and the mean tangent velocity component were computed in the outlet section at each moment of time t_n as the arithmetic means of the corresponding values $p_{N_1-1/2, i,k}^n$ and $(u_t)_{N_1-1/2, i,k}^n$.



Fig. 15. The mean pressure (a) and the tangent velocity component (b) as the functions of time in the outlet section $\xi = s_{\text{max}}$. Solid lines are the results of the three-dimensional computation; dotted lines are the results of the one-dimensional computation by the TVD scheme from Schacht et al. (2002).



Fig. 16. The mean pressure (a) and the tangent velocity component (b) as the functions of time in the outlet section $\xi = s_{\text{max}}$. Solid lines are the results of using formula (9); dotted lines are the results of using the Nikuradse formula (10).

The results of the three-dimensional and one-dimensional computations can be seen from Fig. 15 to agree well with one another. The maximum pressure value p_{out} computed in the one-dimensional computation amounts to 152.245 bar; the corresponding value p_{out} from the three-dimensional computation equals 148.93 bar according to Table 1. Thus, the mathematical model of the three-dimensional flow gives a slightly lower value of the maximum pressure p_{out} than the one-dimensional model.

We also compare in Fig. 16 the results of using two different formulas for the friction factor λ in threedimensional flow computations. It turns out that despite a significant difference in the λ values produced by the both formulas the pressure amplitudes at the upper outlet of the spiral compensator differ little. This can be explained by the fact that the order of smallness of the both values of λ is the same and is about 10^{-2} .

5.2.3. The discontinuous solution case

As was shown by Schacht et al. (2002), in the case of the discontinuous boundary condition (13) at the lower inlet of the spiral channel the fluid flow is discontinuous at t > 0. The frequency of the formation of shock waves is dictated by the frequency of pressure oscillations according to (13).

The results of three-dimensional computations presented in Figs. 17–24 were obtained on the spatial curvilinear grid of $321 \times 12 \times 12$ nodes. The pressure surfaces are shown in Fig. 17 in section $\zeta = 0.5$ for two moments of time $t = 10^4 \tau = 0.01199$ s and $t = 14 \times 10^3 \tau = 0.01679$ s.

We show in Fig. 18 (the upper row of figures) the streamlines in different sections $\xi = \text{const}$ plotted on the basis of the radial velocity component u_r and vertical velocity component w. The density contours in different channel sections are presented in the lower row of figures in Fig. 18.



Fig. 17. The pressure surfaces $p = p(\xi, \eta, 0.5, t)$ at t = 0.01199 s (a) and t = 0.01679 s (b). The volume rate $Q_0 = 6001/$ min, B = 0.08 m, H = 0.0466 m.



Fig. 18. Streamlines and density contours in sections $\xi = \text{const}$ at $t = 14 \times 10^3 \tau = 0.01679 \text{ s}$: (a) $\xi = 0.5\Delta\xi$ (the section near the lower inlet); (b) $\xi = 0.5(s_{\min} + s_{\max})$ (the section in the spiral middle); (c) $\xi = s_{\max} - 0.5\Delta\xi$ (the section near the upper outlet).



Fig. 19. The stagnation surfaces q = 0 at different moments of time: (a) $t = 10^4 \tau = 0.01199$ s; (b) $t = 14 \times 10^3 \tau = 0.01679$ s; (c) $t = 15500\tau = 0.01859$ s.

We show in Fig. 19 the stagnation surfaces q = 0 at different moments of time. The surface q = 0 can be seen to have the shape of a ribbon, as in the continuous flow case; this time the stagnation surface is, however, much shorter than in the case of Fig. 9. With increasing *t*, the stagnation surface moves upwards in the direction of the upper outlet of the compensator.



Fig. 20. The tangential velocity component $u_t = u_t(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.01679 s (b).



Fig. 21. The radial velocity component $u_r = u_r(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.01679 s (b).



Fig. 22. The surfaces $k_{cf} = k_{cf}(\xi, \eta, 0.5)$ for t = 0.01199 s (a) and t = 0.01679 s (b).

Similarly to Figs. 11 and 12 we show in Figs. 20 and 21 the tangential and radial velocity components u_t and u_r . Comparing these two figures we can see as in the smooth flow case that the radial velocity component is about two orders of magnitude smaller than the tangential velocity component u_t . The w component is again very small in comparison with the tangential velocity component u_t ; we do not present the corresponding plots for the purpose of brevity.

The surfaces $k_{cf} = k_{cf}(\xi, \eta, 0.5)$ (see Eq. (22)) presented in Fig. 22 show that the influence of the centrifugal forces is again insignificant.

Fig. 23 shows some numerical results obtained in the case of the discontinuous pressure function (13) at the inlet of the spiral. It can be seen from Fig. 23(b) that the pressure behind the shock wave front gradually drops as the shock propagates upwards along the spiral channel. This means that the compensator ensures the effect of damping the pressure.

The results of a three-dimensional computation of the mean pressure and the mean tangent velocity component u_t in the outlet section of the spiral channel $\xi = s_{max}$ are compared with the results of the one-dimensional computation by the second-order TVD scheme from Schacht et al. (2002) in Fig. 24. The results of the three-dimensional and one-dimensional computations can be seen to be in a good agreement with one another. The maximum pressure value p_{out} computed in the one-dimensional computation amounts to 155.724 bar; the corresponding value p_{out} from the three-dimensional com-



Fig. 23. The numerical results for the pressure in the case $Q_0 = 600 \, \text{l/min}$ in section $\eta = 0.5$, $\zeta = 0.5$: (a) the temporal pressure distribution in the inlet section $\xi = s_{\text{min}}$ according to (13); (b) the pressure distributions along the spiral channel at different moments of time: $t = 14,000\tau = 0.01679 \,\text{s}$ (solid line); $t = 50,000\tau = 0.05996 \,\text{s}$ (dotted line).



Fig. 24. The mean pressure (a) and the tangent velocity component (b) as the functions of time in the outlet section $\xi = s_{max}$. Solid lines are the results of the three-dimensional computation; dotted lines are the results of the one-dimensional computation by the TVD scheme from Schacht et al. (2002).

putation equals 152.416 bar. Thus, the mathematical model of the three-dimensional flow gives as in the smooth flow regime a slightly lower value of the maximum pressure p_{out} than the one-dimensional model.

It can also be seen from Fig. 24 that the discontinuities in the pressure and velocity are smeared in time less intensively when using the TVD scheme despite the fact that we have used only 200 cells along the *x*-axis at the computation by the TVD scheme. This is related to the fact that the TVD scheme used by us in the one-dimensional computations has the second order of accuracy, whereas the Roe's scheme has only the first order of accuracy.

6. Conclusion

We have described above a difference method for the computation of three-dimensional compressible fluid flows in the channels of spiral compensators of the percussion–rotary drilling devices and presented some results of three-dimensional computations. The main purpose of these computations was to gain a further insight into the properties of fluid flow in spiral compensators and to compare the results of three-dimensional computations with the numerical results obtained on the basis of the application of a mathematical model of one-dimensional fluid flow. The above presented results of three-dimensional computations show that the fluid flow under study has such an intrinsically three-dimensional feature as an extended curved stagnation surface q = 0. This surface is much shorter in the case of a shocked flow regime.

Despite this feature, the temporal mean distributions of pressure and tangential velocity component in the upper outlet section of the spiral compensator agree well with the results of one-dimensional computations. This indicates that the influence of the centrifugal forces is not large enough to alter the pressure distribution significantly.

Since the three-dimensional computations require much larger CPU time expenses than the onedimensional computations one can propose the following two-stage procedure for the computational design of optimal spiral compensators:

- a check-up of the proposed compensator design from the viewpoint of its optimality with the aid of one-dimensional computations; the design refinement with the aid of a number of runs using the one-dimensional model;
- the final refinement of the compensator design with the aid of three-dimensional computations.

One can formulate at least two directions of future research in the area of the numerical modelling of fluid flows in spiral compensators.

The first direction follows from the above-mentioned fact that the results of three-dimensional computations averaged over the channel cross section agree well with the results of one-dimensional computations. One can, therefore, perform an averaging over the radial coordinate or over the vertical coordinate in the equations governing the three-dimensional flow and derive in this way the equations of a quasi-twodimensional nonstationary fluid flow in a spiral channel. Since the resulting mathematical model will indeed be a two-dimensional mathematical model its practical application must result in very significant computer time savings at the numerical computations for the purpose of the optimal compensator design.

The second research direction is related to a search for an optimal pressure distribution $p = p_0(t)$ at the lower inlet of the spiral compensator, which will ensure the most efficient damping of pressure peaks in the course of their propagation in the spiral channel for the same peak pressure p_{max} at the lower inlet.

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Appendix. The expressions for matrices R₁, R₂, and R₃

We at first present the expressions for matrices A, B, and C entering formulas (16):

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ c^2 - u^2 & 2u & 0 & 0 \\ -uv & v & u & 0 \\ -uw & w & 0 & u \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ -uv & v & u & 0 \\ c^2 - v^2 & 0 & 2v & 0 \\ -vw & 0 & w & v \end{pmatrix},$$
$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ -uw & w & 0 & u \\ -vw & 0 & w & v \\ c^2 - w^2 & 0 & 0 & 2w \end{pmatrix}.$$

In the expansion $\hat{\mathbf{A}} = J(\xi_x \mathbf{A} + \xi_y \mathbf{B}) = \mathbf{R}_1 \Lambda_1 \mathbf{R}_1^{-1}$, Λ_1 is a diagonal matrix whose diagonal entries are the eigenvalues of matrix $\hat{\mathbf{A}}$, and the multiple eigenvalues are repeated on the diagonal in accordance with their multiplicity. For the computation of matrices \mathbf{R}_1 and Λ_1 in the expansion of matrix $\hat{\mathbf{A}}$ one can use the following built-in functions of the software system *Mathematica*, respectively: Eigenvalues[\mathbf{A}] and Transpose[Eigenvectors[\mathbf{A}]]. There is also the built-in function JordanDecomposition[\mathbf{A}], which outputs both \mathbf{R}_1 and Λ_1 . Therefore, we have used this function in view of its convenience: $[\mathbf{R}_1, \Lambda_1] =$ **JordanDecomposition**[\mathbf{As}], where $\mathbf{As} \equiv \hat{\mathbf{A}}$, and $\Lambda_1 = \text{diag}(\lambda_1^{(1)}, \lambda_2^{(1)}, \lambda_3^{(1)}, \lambda_4^{(1)}), \lambda_1^{(1)} = \lambda_2^{(1)} = J\hat{u}, \lambda_3^{(1)} =$ $\lambda_1^{(1)} - Jc|\nabla\xi|, \lambda_4^{(1)} = \lambda_1^{(1)} + Jc|\nabla\xi|, |\nabla\xi| = \sqrt{\xi_x^2 + \xi_y^2}, \hat{u} = \xi_x u + \xi_y v$. As a result, an expression for matrix $\tilde{\mathbf{R}}_1$ was found, in which there is the velocity component w in the denominators of some entries. Thus, a singularity arises in these entries at w = 0. On the other hand it is well known that the matrix $\tilde{\mathbf{R}}_1$ in the Jordan decomposition is determined nonuniquely. Let indeed \mathbf{D}_1 be a nonsingular diagonal matrix, and let $\mathbf{R}_1 = \tilde{\mathbf{R}}_1 \Lambda_1 \tilde{\mathbf{R}}_1^{-1} = \hat{\mathbf{A}}$. If we choose the matrix \mathbf{D}_1 as $\mathbf{D}_1 = \text{diag}(c, c\xi_x, w, w)$, then we obtain:

$$\mathbf{R}_{1} = \tilde{\mathbf{R}}_{1} \mathbf{D}_{1} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & -c\xi_{y} & -c\hat{\xi}_{x} + u & c\hat{\xi}_{x} + u \\ 0 & c\xi_{x} & -c\hat{\xi}_{y} + v & c\hat{\xi}_{y} + v \\ c & 0 & w & w \end{pmatrix},$$

$$\mathbf{R}_{1}^{-1} = \begin{pmatrix} -\frac{w}{c} & 0 & 0 & \frac{1}{c} \\ \frac{u\xi_{y} - v\xi_{x}}{c|\nabla\xi|^{2}} & \frac{-\xi_{y}}{c|\nabla\xi|^{2}} & \frac{\xi_{x}}{c|\nabla\xi|^{2}} & 0 \\ \frac{1}{2}\left(1 + \frac{\hat{\xi}_{x}u + \hat{\xi}_{y}v}{c}\right) & -\frac{\hat{\xi}_{x}}{2c} & -\frac{\hat{\xi}_{y}}{2c} & 0 \\ \frac{1}{2}\left(1 - \frac{\hat{\xi}_{x}u + \hat{\xi}_{y}v}{c}\right) & \frac{\hat{\xi}_{x}}{2c} & \frac{\xi_{y}}{2c} & 0 \end{pmatrix},$$
(23)

where $\hat{\xi}_x = \xi_x / |\nabla \xi|, \hat{\xi}_y = \xi_y / |\nabla \xi|.$

Similarly, the Jordan decomposition of the matrix $\hat{\mathbf{B}} = J(\eta_x \mathbf{A} + \eta_y \mathbf{B}) = \mathbf{R}_2 \Lambda_2 \mathbf{R}_2^{-1}$, where

$$\begin{split} \Lambda_2 &= \operatorname{diag}(\lambda_1^{(2)}, \lambda_2^{(2)}, \lambda_3^{(2)}, \lambda_4^{(2)}), \quad \lambda_1^{(2)} = \lambda_2^{(2)} = J\hat{v}, \quad \lambda_3^{(2)} = \lambda_1^{(2)} - Jc|\nabla\eta|, \\ \lambda_4^{(2)} &= \lambda_1^{(2)} + Jc|\nabla\eta|, \\ |\nabla\eta| &= \sqrt{\eta_x^2 + \eta_y^2}, \quad \hat{v} = \eta_x u + \eta_y v, \end{split}$$

$$\mathbf{R}_{2} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & -c\eta_{y} & -c\hat{\eta}_{x} + u & c\hat{\eta}_{x} + u \\ 0 & c\eta_{x} & -c\hat{\eta}_{y} + v & c\hat{\eta}_{y} + v \\ c & 0 & w & w \end{pmatrix},$$

$$\mathbf{R}_{2}^{-1} = \begin{pmatrix} -\frac{w}{c} & 0 & 0 & \frac{1}{c} \\ \frac{u\eta_{y} - v\eta_{x}}{c|\nabla\eta|^{2}} & -\frac{\eta_{y}}{c|\nabla\eta|^{2}} & \frac{\eta_{x}}{c|\nabla\eta|^{2}} & 0 \\ \frac{1}{2}\left(1 + \frac{\hat{\eta}_{x}u + \hat{\eta}_{y}v}{c}\right) & -\frac{\hat{\eta}_{x}}{2c} & -\frac{\hat{\eta}_{y}}{2c} & 0 \\ \frac{1}{2}\left(1 - \frac{\hat{\eta}_{x}u + \hat{\eta}_{y}v}{c}\right) & \frac{\hat{\eta}_{x}}{2c} & \frac{\hat{\eta}_{y}}{2c} & 0 \end{pmatrix},$$

where $\hat{\eta}_x = \eta_x / |\nabla \eta| = \cos \xi$, $\hat{\eta}_y = \eta_y / |\nabla \eta| = -\sin \xi$. And, finally,

$$\hat{\mathbf{C}} = J(\zeta_x \mathbf{A} + \zeta_y \mathbf{B} + \zeta_z \mathbf{C}) = \mathbf{R}_3 \Lambda_3 \mathbf{R}_3^{-1},$$

where

$$\begin{split} &\Lambda_{3} = \operatorname{diag}(\lambda_{1}^{(3)}, \lambda_{2}^{(3)}, \lambda_{3}^{(3)}, \lambda_{4}^{(3)}), \quad \lambda_{1}^{(3)} = \lambda_{2}^{(3)} = J\hat{w}, \quad \lambda_{3}^{(3)} = \lambda_{1}^{(3)} - Jc|\nabla\zeta|, \\ &\lambda_{4}^{(3)} = \lambda_{1}^{(3)} + Jc|\nabla\zeta|, \\ &|\nabla\zeta| = \sqrt{\zeta_{x}^{2} + \zeta_{y}^{2} + \zeta_{z}^{2}}, \quad \hat{w} = \zeta_{x}u + \zeta_{y}v + \zeta_{z}w, \\ &\mathbf{R}_{3} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ -\frac{c\zeta_{z}}{\zeta_{x}} & -\frac{c\zeta_{y}}{\zeta_{x}} & -c\hat{\zeta}_{x} + u & c\hat{\zeta}_{x} + u \\ 0 & c & -c\hat{\zeta}_{y} + v & c\hat{\zeta}_{y} + v \\ c & 0 & -c\hat{\zeta}_{z} + w & c\hat{\zeta}_{z} + w \end{pmatrix}, \\ &\mathbf{R}_{3}^{-1} = \begin{pmatrix} b_{11} & -\frac{\hat{\zeta}_{x}\hat{\zeta}_{z}}{c} & -\frac{\hat{\zeta}_{y}\hat{\zeta}_{z}}{c} & \frac{\hat{\zeta}_{x}^{2} + \hat{\zeta}_{z}^{2}}{c} \\ b_{21} & -\frac{\hat{\zeta}_{x}\hat{\zeta}_{y}}{c} & \frac{\hat{\zeta}_{x}^{2} + \hat{\zeta}_{z}^{2}}{c} & -\frac{\hat{\zeta}_{y}\hat{\zeta}_{z}}{c} \\ \frac{1}{2} + \frac{\hat{w}}{2c|\nabla\zeta|} & -\frac{\hat{\zeta}_{x}}{2c} & -\frac{\hat{\zeta}_{y}}{2c} & -\frac{\hat{\zeta}_{z}}{2c} \\ \frac{1}{2} - \frac{\hat{w}}{2c|\nabla\zeta|} & \frac{\hat{\zeta}_{x}}{2c} & \frac{\hat{\zeta}_{y}}{2c} & \frac{\hat{\zeta}_{z}}{2c} \end{pmatrix}. \end{split}$$

Here $\hat{\zeta}_x = \zeta_x/|\nabla\zeta|, \hat{\zeta}_y = \zeta_y/|\nabla\zeta|, \hat{\zeta}_z = \zeta_z/|\nabla\zeta|, b_{11} = [\hat{\zeta}_z(\hat{\zeta}_x u + \hat{\zeta}_y v) - (\hat{\zeta}_x^2 + \hat{\zeta}_y^2)w]/c, b_{21} = [\hat{\zeta}_x \hat{\zeta}_y u - \hat{\zeta}_x^2 v + \hat{\zeta}_z (\hat{\zeta}_y w - \hat{\zeta}_z v)]/c.$ Note that $\text{Det}(\mathbf{R}_1) = -2c^3|\nabla\xi| < 0$, $\text{Det}(\mathbf{R}_2) = -2c^3|\nabla\eta| < 0$, $\text{Det}(\mathbf{R}_3) = -2c^3|\nabla\zeta| < 0$. In the case of the vanishing ζ_x , the following expressions were used for \mathbf{R}_3 and \mathbf{R}_3^{-1} :

$$\mathbf{R}_{3} = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & c & u & u \\ c\zeta_{z} & 0 & -c\hat{\zeta}_{y} + v & c\hat{\zeta}_{y} + v \\ -c\zeta_{y} & 0 & -c\hat{\zeta}_{z} + w & c\hat{\zeta}_{z} + w \end{pmatrix},$$

$$\mathbf{R}_{3}^{-1} = \begin{pmatrix} \frac{\zeta_{z}v - \zeta_{y}w}{c|\nabla\zeta|^{2}} & 0 & \frac{-\zeta_{z}}{c|\nabla\zeta|^{2}} & \frac{\zeta_{y}}{c|\nabla\zeta|^{2}} \\ \frac{-u}{c} & \frac{1}{c} & 0 & 0 \\ \frac{1}{2}\left(1 + \frac{\hat{\zeta}_{y}v + \hat{\zeta}_{z}w}{c}\right) & 0 & -\frac{\hat{\zeta}_{y}}{2c} & \frac{-\hat{\zeta}_{z}}{2c} \\ \frac{1}{2}\left(1 - \frac{\hat{\zeta}_{x}v + \hat{\zeta}_{z}w}{c}\right) & 0 & \frac{\hat{\zeta}_{y}}{2c} & \frac{\hat{\zeta}_{z}}{2c} \end{pmatrix}$$

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